Sec. 4.5 The Number e

An irrational number, introduced by Euler in 1727, is so important that it is given a special name, *e*. Its value is approximately e ≈ 2.71828 . . .. It is often used for the base, b, of the exponential function. Base *e* is called the natural base. This may seem mysterious, as what could possibly be natural about using an irrational base such as *e*? The answer is that the formulas of calculus are much simpler if *e* is used as the base for exponentials.

For the exponential function $Q = a\,b^t$, the **continuous growth rate, *k*,** is given by solving $e^k = b$. Then

$$Q = a\,e^{kt}.$$

If a is positive,

- If k > 0, then Q is increasing.
- If k < 0, then Q is decreasing.

Continuous compounding – The amount A after t years due to a principal P invested at an annual interest rate compounded continuously is:

$$A = Pe^{rt}.$$

Ex. On January 2, 2002, $2000 is placed in an IRA that will pay interest of 12% per annum compounded continuously. What will the IRA be worth on January 1, 2022? What is the effective rate of interest?

$A = Pe^{rt}$
$= 2000 \cdot e^{.12(20)}$
$A = \$22{,}046.35$

$e^{.12(1)} = 1.1275$
$er = 12.75\%$

Ex. 1 Give the continuous growth rate of each of the following functions and graph each function:

$$P = 5e^{0.2t}, \quad Q = 5e^{0.3t}, \quad \text{and} \quad R = 5e^{-0.2t}.$$

20% growth   30% growth   -20% growth

Ex. 2 Caffeine leaves the body at a continuous rate of 17% per hour. How much caffeine is left in the body 8 hours after drinking a cup of coffee containing 100 mg of caffeine?

$100\,e^{-.17(8)}$
25.67mg

Ex. 3 In November 2005, the Wells Fargo Bank offered interest at a 2.323% continuous yearly rate. Find the effective annual rate.

$e^{.02323(1)} = 1.0235$ $\quad r = 2.35\%$

Ex. 4 Which is better: An account that pays 8% annual interest compounded quarterly or an account that pays 7.95% annual interest compounded continuously?

$A = P\left(1+\frac{.08}{4}\right)^{4(1)}$ $\quad A = Pe^{.0795(1)}$

$A = 1.0824P$ $\quad A = 1.0827P$

7.95% Compounded Continuously has a higher effective rate

Ex. How long will it take for an investment to double in value if it earns 5% compounded continuously?

$A = Pe^{rt}$

$2P = Pe^{rt}$

$2 = e^{.05t}$ Graph

$t = 13.86$ years

HW: pg 163-167, # 3, 6, 14, 17, 20, 23, 26, 28 ,31, 33, 35, 36, 38, choose 3 from 40-51